On the Estimation of Disability-Free Life Expectancy

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- DFLE equals,

$$e^{DF}(x, y) = \frac{1}{l(x, y)} \int_x^\infty [1 - \pi(t, y)]\ l(t, y)\, dt.$$

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- Sullivan's estimator and variance of Sullivan's estimator:

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$$\hat{\sigma}_x^{DF} = \frac{1}{l_x^2} \sum_{i \in A_x} \frac{{}_{n_i}\hat{\pi}_i\ (1 - {}_{n_i}\hat{\pi}_i)\ {}_{n_i}L_i^2}{{}_{n_i}N_i}$$

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  - ⋆ Recovery transitions (Barendregt et al. 1994,95; Van De Water et al. 1995).

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- Conflicting results over required assumptions:
  - ⋆ Probability of transition (healthy to disabled) 'large' (Newman 1988).
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- Our Theoretical results:
  - ⋆ Sullivan's method does not make any assumption about the transition probabilities other than that these probabilities must be stationary.
  - ⋆ Sullivan's method does not make any assumptions regarding the homogeneity of mortality risk between the healthy and disabled.

Assumptions and Statistical Properties of Sullivan's Method

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- Proposition 1. Suppose that the three stationary assumptions of period life tables hold. In addition, suppose that the age-specific disability prevalence is constant over time, i.e., $\pi(x, y) = \pi(x)$ for all y. Then, Sullivan's method estimates DFLE without bias, i.e., $E(\hat{e}_x^{DF}) = e^{DF}(x)$.

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- Proposition 2. Under the four stationarity assumptions of Proposition 1, the variance of Sullivan's estimator is given by,

$$\sigma_x^{DF} = \frac{1}{l_x^2} \sum_{i \in A_x} \frac{E[\pi(s)]\{1 - E[\pi(s)]\}\, {}_{n_i}L_i^2}{{}_{n_i}N_i}.$$

  The standard variance estimator consistently estimates σ_x^{DF} without any assumption about the function form of $\pi(x)$.

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- The unbiased estimation of DFLE is also possible with consecutive cross-sectional disability surveys — easier to obtain than longitudinal data.

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- Sullivan's estimator of DFLE for the cohort born in year y is defined by,

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$$\{1 - E[\pi(s,y)]\} \; {}_nL_{x-n} \leq \int_{x-n}^{x} [1 - \pi(t,y)] \; l(t,y) \, dt \;\; \leq \;\; {}_nL_{x-n,y}.$$

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$$0 \leq \int_{\omega^*+n_{\omega^*}}^{\infty} [1 - \pi(t,y)] \; l(t,y) \, dt \;\; \leq \; \{1 - E[\pi(s,y)]\} \, {}_\infty L_{\omega^*+n_{\omega^*},y}.$$

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- Self-Reported Disability: at least one ADL.

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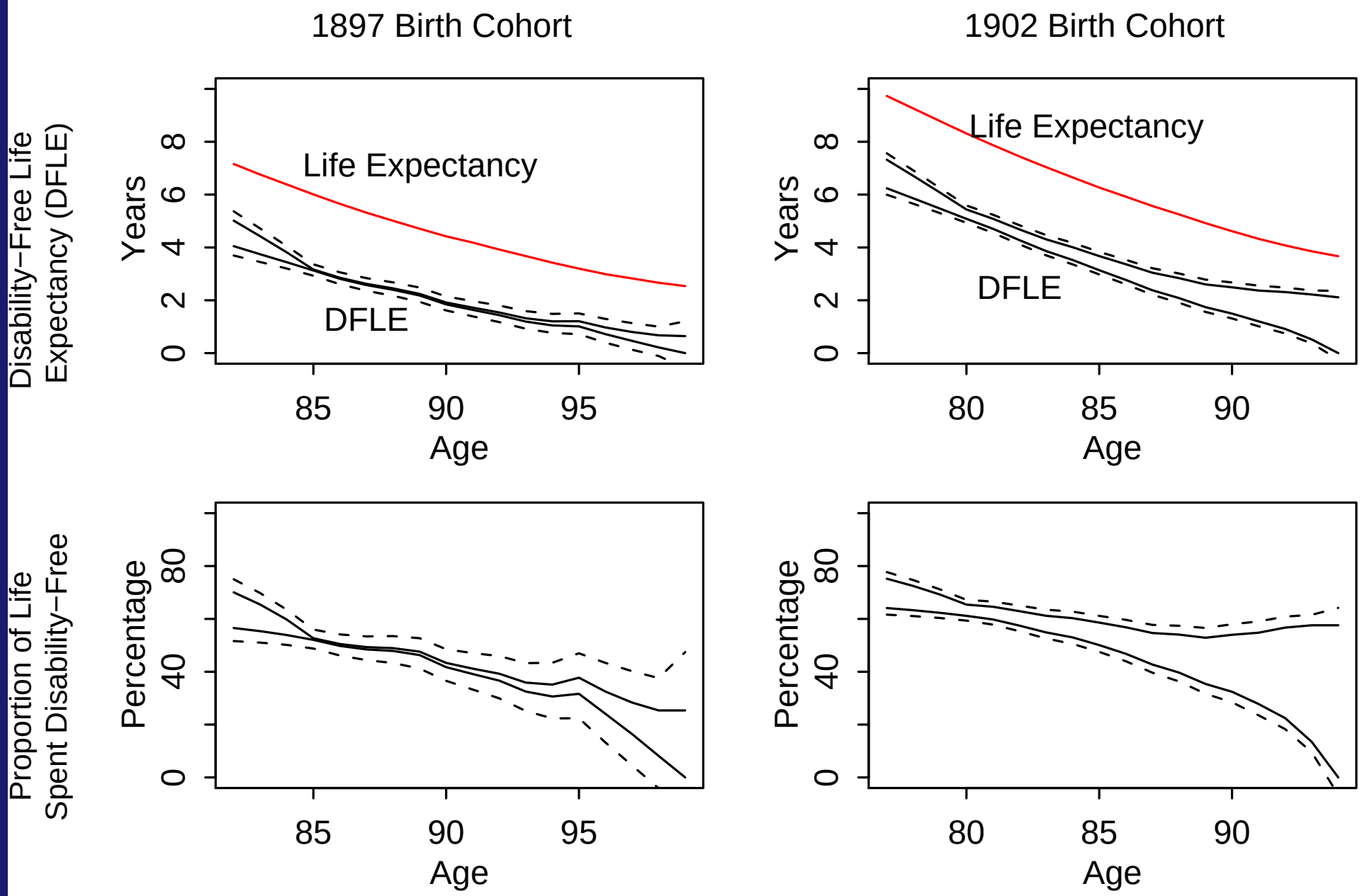
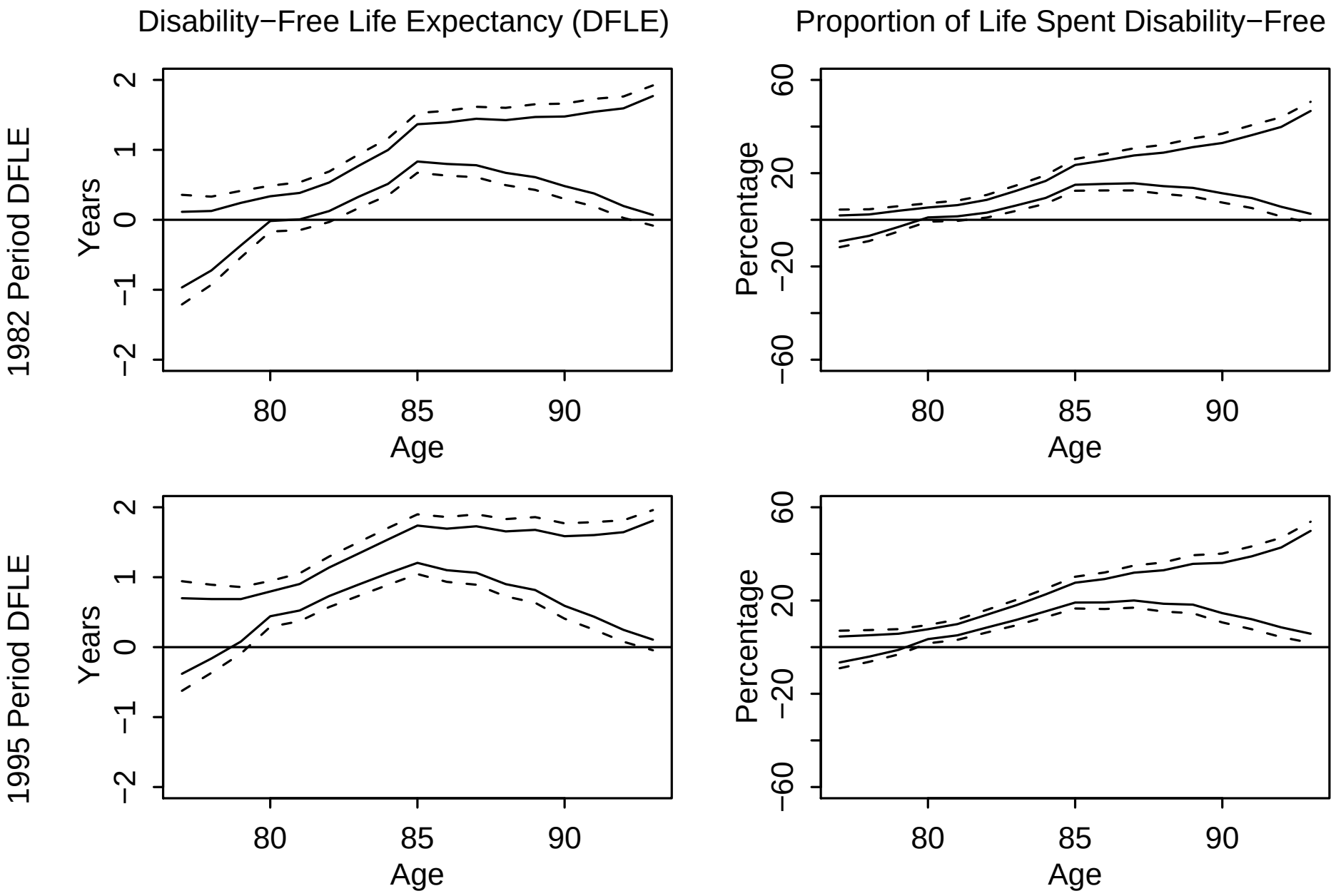
- Race: White and Black, without respect to Hispanic ethnicity.

	Mortality Data		Disability Data	
Birth Cohort	From	To	From	To
1897	91	106	94	106
1902	86	101	89	101
1914	74	89	77	89

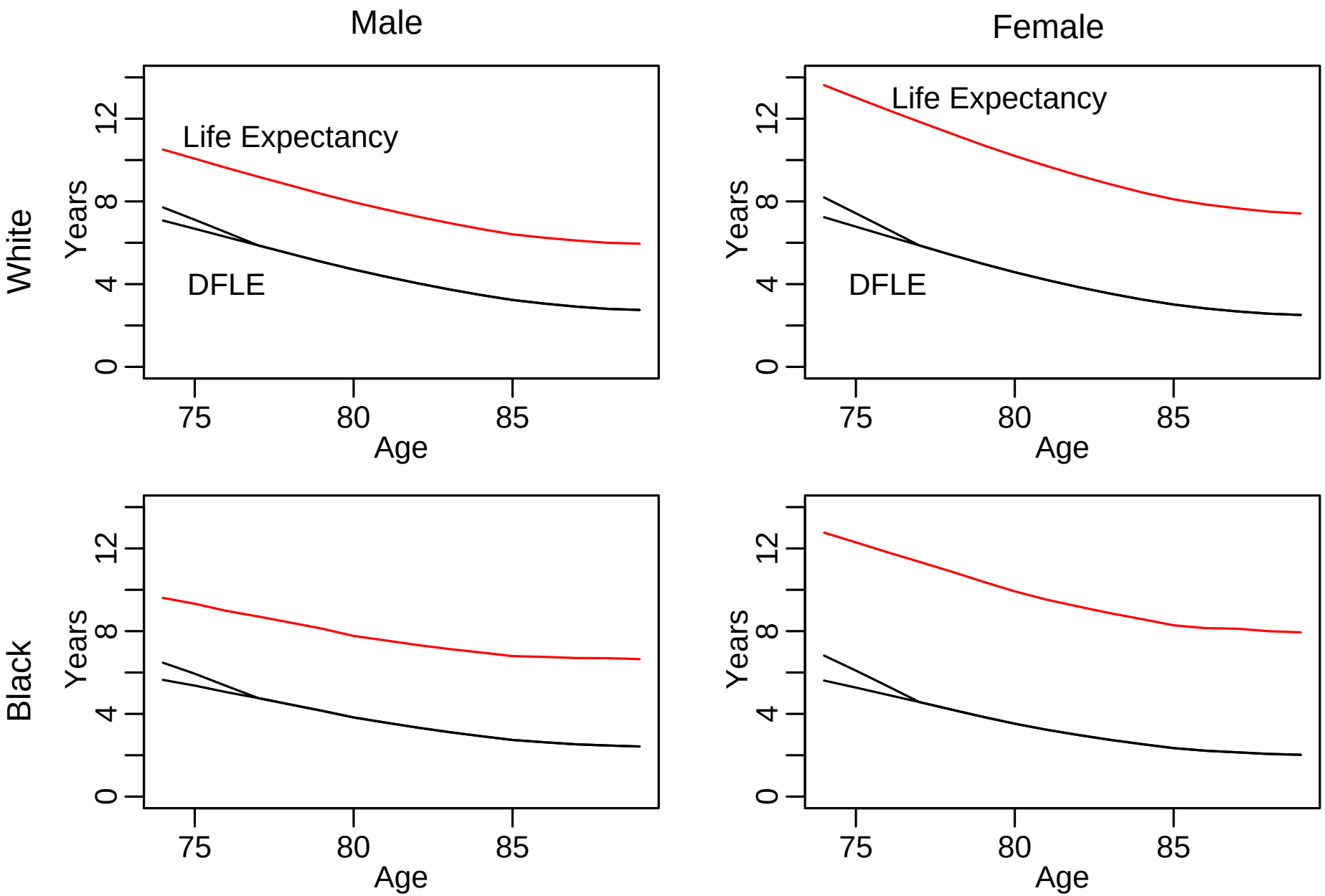
Estimated DFLE and Proportion of Life Spent Disability-Free



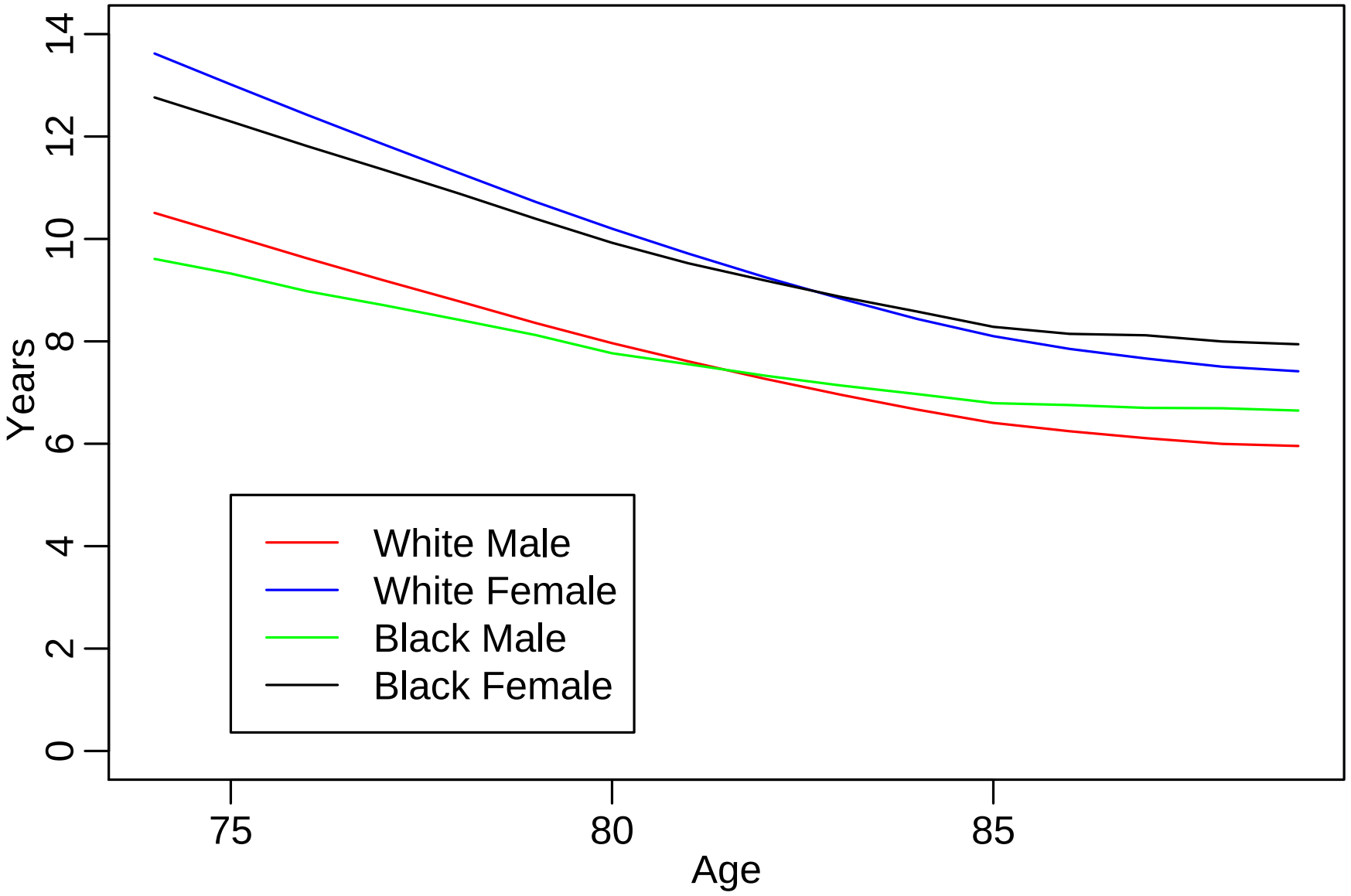
Cohort (1902 Birth Cohort) Versus Period DFLE



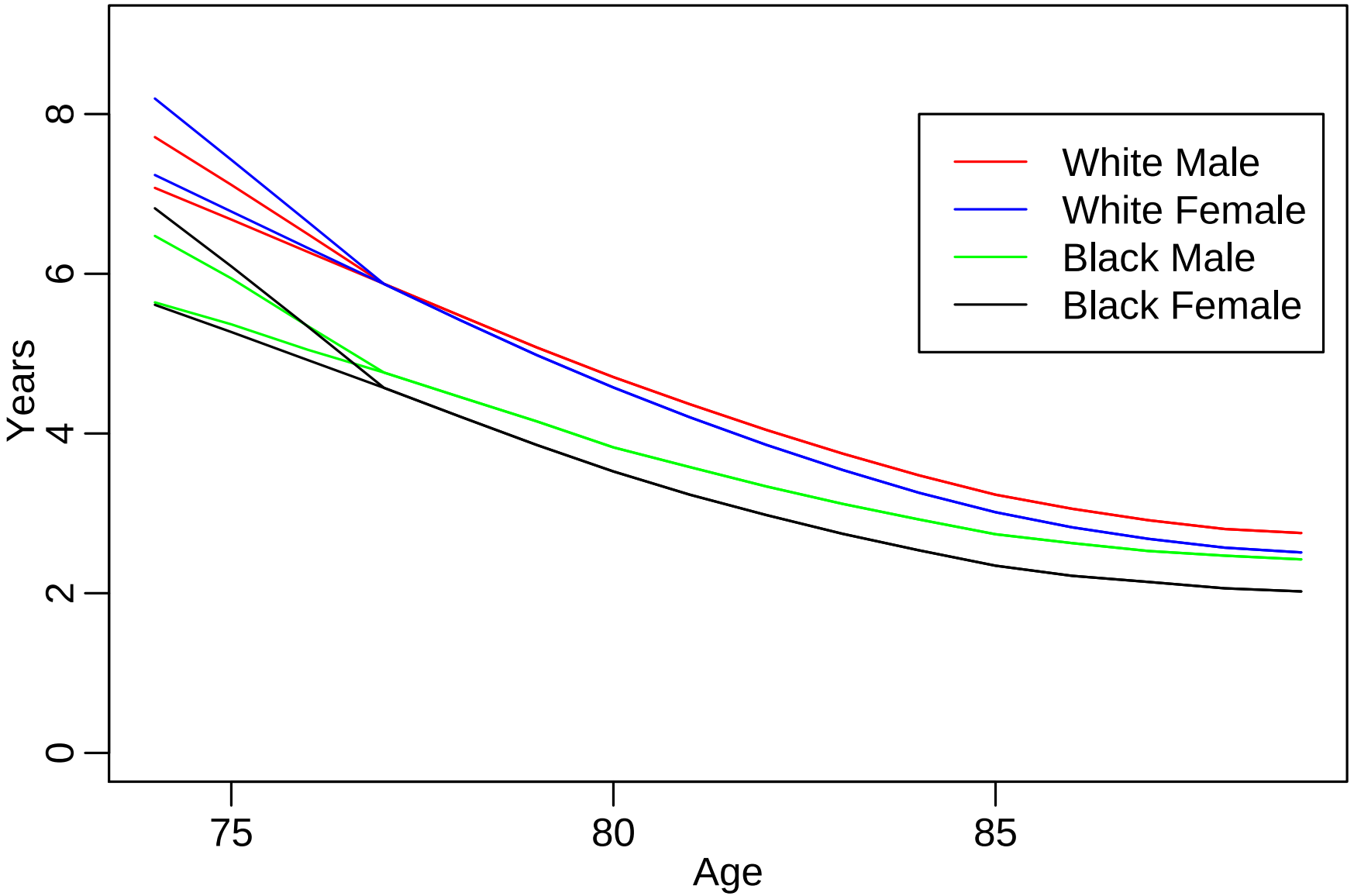
Sex and Racial Disparities of the 1914 Birth Cohort



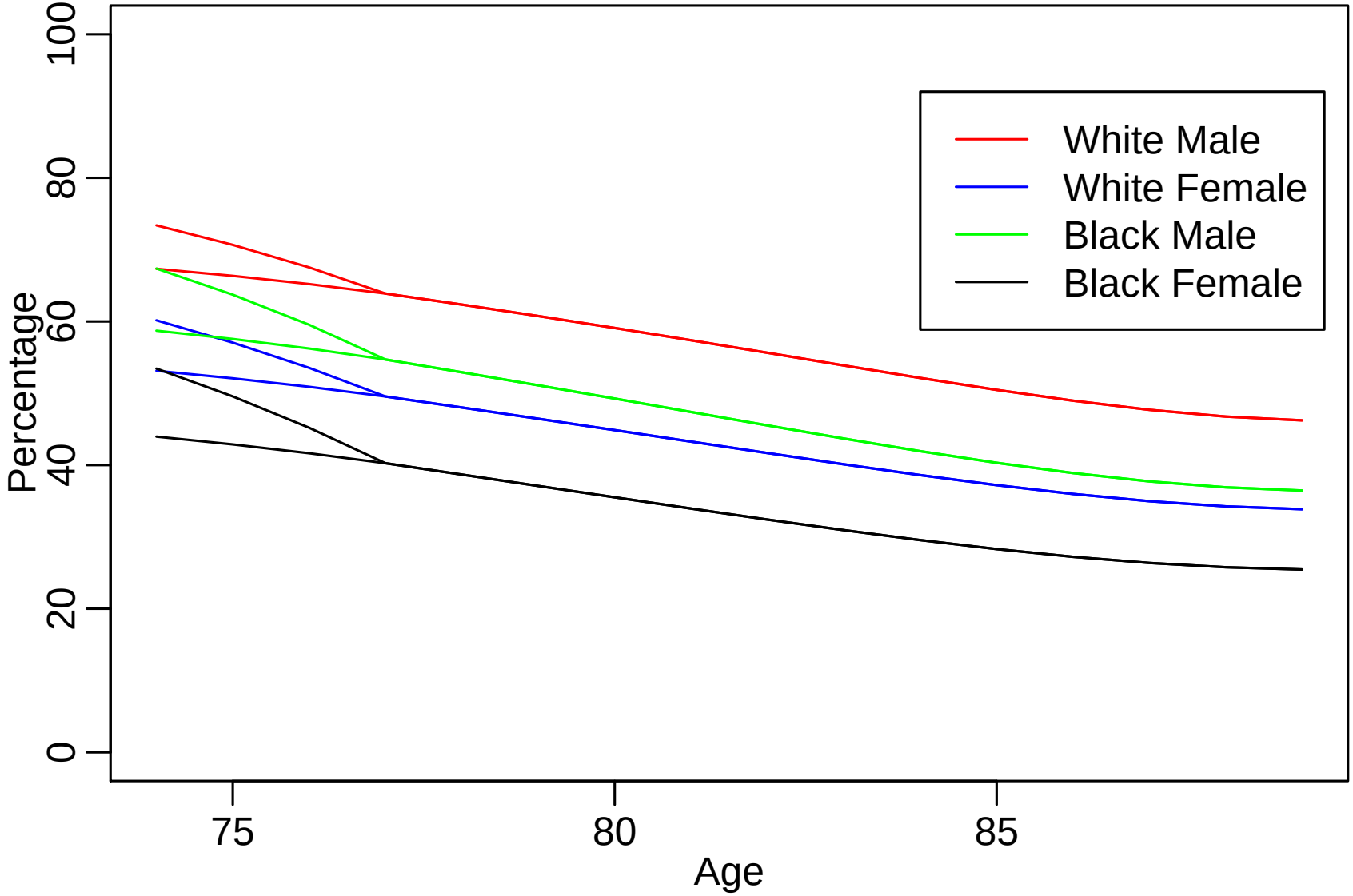
Life Expectancy, 1914 Birth Cohort



Disability-Free Life Expectancy, 1914 Birth Cohort



Proportion of Remaining Life Spent Disability-Free, 1914 Birth Cohort



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- Non-stationary mortality and disability yields significant cohort and period differences in DFLE, especially at older ages.
- While sex-specific mortality may exhibit a 'crossover' between races, DFLE and proportion of remaining life spent disability-free may not.