Health Expectancy Calculation by the Sullivan Method: A Practical Guide

European Concerted Action on the Harmonization of Health Expectancy Calculations in Europe (EURO-REVES)[*]

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This work grew out of the needs of the sub-committee on mental health of Euro-REVES in collaboration with the sub-committee on research design and calculation methods. Euro-REVES is a European Concerted Action on the Harmonization of Health Expectancy Calculations in Europe. The main aims of Euro-REVES are:

1. To develop guidelines for the construction and calculation of health expectancy indices with a view to European harmonization of concepts, data collection and calculation methods.
2. To provide a European data base[1] on health expectancy estimators, accessible to individuals and organizations involved in health research and development.
3. To produce a reference document describing concepts, questionnaires and calculation methods which may be used as a basis for the further promotion of calculations both within and outside Europe.

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[1] The World-Wide server can be accessed at the following address http://euroreves.ined.fr/euroreves and is maintained by Nicolas Brouard (Brouard@sauvy.ined.fr)

Contents

Summary

What is Health Expectancy?
Life expectancy is composed of lengths of time spent in different states of health until death. These lengths of time in different states of health are health expectancies and they combine information on both mortality and morbidity.

What is Health Expectancy by Sullivan's method?
The concept of health expectancies as health indicators was proposed by Sanders (1964) and the first example was published in a report of the US Department of Health Education and Welfare (Sullivan, 1971). This report contained preliminary estimates of "Disability-Free Life Expectancy" calculated using a method devised by Sullivan and applicable to any state of health definition. We shall use the term Sullivan Health Expectancy as a shorthand for health expectancy calculated by the Sullivan method.

The Sullivan health expectancy reflects the current health of a real population adjusted for mortality levels and independent of age structure. Health expectancy calculated by Sullivan's method is the number of remaining years, at a particular age, that an individual can expect to live in a healthy state (however health may be defined). For example, in 1991 in Great Britain, women aged between 65 and 69 years of age could expect to live a further 18.1 years of which 16.9 years (93%) would be spent without disability, disability being defined as the inability to perform activities of daily living (Bone et al., 1995).

What information is required to calculate the Sullivan health expectancy?
The data required are the age-specific prevalence (proportions) of the population in healthy and unhealthy states (often obtained from cross-sectional surveys), and age-specific mortality information taken from a period life table. The Sullivan health expectancy is not very sensitive to the size of the age groups, meaning that an abridged life table[2] may be used. Generally, it is preferable to use five (sometimes ten)-year age intervals because most surveys used to derive the age-specific proportions (prevalence) in healthy and unhealthy states are too small to allow smaller age intervals. This is particularly important at the higher ages and it is usual to include the final age interval as 85 years and over.

What is the Sullivan health expectancy used for?
The Sullivan health expectancy provides a means of comparing the health states of an entire population at two time points or of two different populations at the same time point, despite any differences in age composition (provided that the age intervals are not too large). The same definitions of health states and age intervals must be used for the populations and/or time points being compared. Comparability is also increased by the calculation of this indicator separately for males and females and if necessary (to adjust for structural differences) for other categories.

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[2] An abridged life table is like a complete life table but has age intervals greater than one year (apart from the first years of life). A typical set of intervals might be 0 to under 1 year, 1 to under 5 years, 5 to under 10 years, 10 to under 15 years, etc.

If comparisons are to be made between several measures of health expectancies, a number of provisos need to be made.

- It is important that the same definitions of health states are used for each of the health expectancies. For example, differences between health expectancies calculated for different countries can often be explained by differences in the measurement instruments used to collect the prevalence data (Boshuizen and van de Water, 1994).
- The general framework of the surveys used to derive prevalance also need to be the same to allow comparisons. The estimates of the prevalence of ill-health are very sensitive to the way the data are collected (eg face to face interview, telephone interview, postal questionnaire).
- Some surveys exclude institutionalized persons which may produce bias particularly for older populations and certain health conditions (Ritchie et al., 1992). It may be possible to incorporate data from separate surveys of the institutionalized population although this often requires strong assumptions to be made (see Example 5).

Formal statistical comparisons may be made between times or place as the standard errors of estimates are relatively simple to calculate. To date, Sullivan health expectancies have provided health indicators on populations of at least 37 countries (Robine et al., 1995) with increasing interest being shown from policy makers (van de Water, 1992).

What are the differences between the Sullivan method and other methods to calculate health expectancy?

To calculate health expectancy at a particular age and time, it is necessary to calculate the person years lived in the health state from that age at the particular time. Thus, theoretically, estimates of health expectancies at this time depend on the incidence of health states and are essentially longitudinal measures. Direct calculation of the person years lived in the health state requires longitudinal data to provide the transition rates between health states (multistate method). The Sullivan method is of particular practical importance as it uses more readily available data, age-specific prevalence of the health state and the total person years lived at a particular age. Obviously there must be some error associated with this approximation (except if all population characteristics are stable in time) but experience has shown that the Sullivan method can, generally, be recommended for its simplicity, relative accuracy and ease of interpretation.

In particular, the Sullivan and multistate methods seem to produce similar results providing all transition rates are smooth and regular over time (Robine and Mathers, 1992). When prevalence remains the same between two periods whereas incidence rates between states of health change rapidly then the Sullivan method may underestimate (or overestimate) health expectancy, because the prevalence of ill health at a given age in the population reflect the past probabilities of becoming ill at each younger age (Mathers, 1991). In this case though, the Sullivan health expectancy remains a meaningful indicator of the state of health at a population, rather than prediction at an individual, level. Differences between the multistate and Sullivan indicators also occur if prevalence changes (e.g. because of a decrease in mortality) whilst incidence remains constant (Barendregt et al., 1994). Many researchers have commented on the differences between the Sullivan and multistate methods (Rogers, Rogers and Branch, 1990; Robine and Ritchie, 1991; Bebbington, 1991; Mathers, 1991; van de Water et al., 1995).

The remainder of this book provides easy-to-follow examples of health expectancy calculations using the Sullivan method. Readers who require more technical explanations of the issues underlying these calculations will find references at the relevant points to sections in Appendix 1. The first example uses a single year life table whilst the second example shows the difference when an abridged life table is used. The third example documents the calculation of the standard errors and confidence interval for the health expectancy whilst the fourth example tests the equality of two health expectancies either from two points in time. Finally, the fifth example demonstrates how to calculate health expectancy when the age-specific prevalence of the health state are obtained from community-living and institutionalized persons separately. The first three examples are also available on a Microsoft Excel spreadsheet accompanying this manual as well as on the Internet (downloaded from ftp://euroreves.ined.fr/euroreves/methods/sulliv2.xls). The spreadsheet also has the ability to input other data values and to explore the effect of changing sample sizes on the standard errors of the indicator.

Sullivan health expectancy using an unabridged life table: Example 1

To make the calculation easier to follow, we have chosen to begin the life table at age 75 years rather than birth. However the calculations would follow in exactly the same way with the full age range (0 to 90+ years). These calculations are presented in tabular form in Tables 1.1-1.3 and are easily performed on a spreadsheet (e.g. Microsoft Excel, QuattroPro or Lotus 1-2-3). The health expectancy that we shall calculate is disability-free life expectancy for women in the UK. Disability will be defined as requiring help with one or more activities of daily living (ADLs). Recall that the data required for such calculations are morbidity data (in the form of age-specific prevalence) and mortality data (from a period life table).

Morbidity data
The prevalence of disability by sex and age group (75-79 years, 80-84 years, 85+ years) are taken from a survey conducted during 1988 in a total general practice population. The survey included people living at home as well as in institutional care. Further details of the survey can be found in Bone et al. (1995)[3]. The prevalence of disability for the 75-79 year age group was 0.140 (66/473), for those aged 80-84 years 0.251 (85/339) and for the 85 year olds and over 0.559 (128/229). Since the study is small, the prevalence will be tabulated in 5 year age groups and then attributed to each of the single years of age. See Example 2 for another method of analysis using an abridged life table.

Mortality data
The most important quantities for calculation of life (and therefore health) expectancy are the person years lived in each age group by a future cohort assuming the same age-specific mortality rates apply. To calculate these we need to know the total time spent in each age group by each member of the cohort. These data are not available as we do not usually have each individual's life history. Instead we can estimate these using the population in each age group and the number of deaths in the age group. The figures for population and deaths are usually obtainable in published form from National Statistics Offices. In the case of the UK they are published annually by the Office of National Statistics (ONS), formally the Office of Population, Censuses and Surveys (OPCS).

Since the morbidity data (prevalence) relate to 1988, we need to use mortality data for the same period. In Tables 1.1 - 1.3 following, the data that is input (eg mortality and morbidity data) are shown in italics. These correspond to columns shown in blue in Table 1 on the Excel spreadsheet accompanying the manual. Table 1.1 shows the data required, these being the mid-year population estimates (column[2]) and the number of deaths by single years of age for women in England and Wales in 1988 (column[3]) and the prevalence of disability (column[10]).

Calculation of the life table and life expectancy
The results of the steps below to calculate the life table and total life expectancy are shown in Table 1.2.

1. The first calculation from the mortality data are the central death rates, M_x (column[4]). Since we have mid-year population estimates (column[2]) and the number of deaths is within one calendar year, the central death rates are calculated as the number of deaths divided by the total

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[3] Also included in Bone et al. (1995) are the figures for disability-free life expectancy as defined in this example. However in Bone et al. the deaths in the three years 1987-1989 are used in the calculation. For simplicity in this example mortality data for 1988 only are used so that final figures for disability-free life expectancy will differ slightly.

population (column[3]/column[2])[4]. If we had had total deaths over a three year period, say 1987-1989 and the mid-interval population (in 1988) then the central death rates would have been calculated as number of deaths divided by the calendar person years (which might be approximated by the mid-interval population multiplied by 3).

2. From the death rates are calculated the probabilities of death in the age interval conditional on having survived to that age, Q_x (column[5]). By definition the final entry in column[5] corresponding to 90+ years is 1 (if a person has survived to 90 years of age then they must die sometime in this interval). The other entries are calculated as column[4]/(1+0.5*column[4])[5].

3. The third quantity is the probability of surviving to each age l_x (column[6]). We assume a starting probability of 1.000 at age 75 years. We then calculate each successive row in column[6], corresponding to the next age interval, as the previous entry multiplied by its corresponding (1-column[5])[6] . For example, if the probability of surviving to the beginning of the age interval 75 is 1.00 (given that our total population is all aged 75 years and over) and the probability of dying in the interval is 0.037 then we would expect the probability of surviving to the beginning of the interval 76 years is 1.000*(1-0.037) = 0.963. Demographers often equate the probability of surviving to the numbers of survivors, multiplying the starting probability (1.0) by an arbitrary "radix population" of 100,000 people and neglecting the random variation of the process.

4. The number of person years lived in the age interval, L_x, is calculated in column[7]. At any age (other than the age interval 90+ years) this is the average of the entry in column [6] for the same age and the subsequent age interval[7]. Thus the number of person years lived in the 75 year age interval is 0.5*(1.000+0.963) = 0.982. If a person survives the age interval then they contribute 1.0 person year. At age 90+ years the person years lived is the number of survivors to age 90 years divided by the death rate (0.268/0.230)[8].

5. Column[8] contains the total number of years lived from the particular age, T_x. This column is calculated by summing all the entries in column[7] from that age to age 90+ years. Hence the total number of years lived from age 86 is given by (0.447+0.393+0.342+0.293+1.167) = 2.641.

6. Finally the total life expectancy at each age e_x (column[9]) is found by dividing the total number of years lived from that age (column[8]) by the probability of surviving to that age (column[6])[8]. Thus in 1988 a woman aged 80 years in England and Wales could expect to live a further 8.161 years.

Calculation of Disability-Free Life Expectancy (DFLE)
Table 1.3 shows the final steps below to calculate DFLE.

7. Disability-Free Life Expectancy (DFLE) is found by partitioning the person years lived in the interval into those lived with and without disability. The prevalence of disability by age is found

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[4] See Appendix 1 paragraph 2. for the general formula.
[5] See Appendix 1 paragraph 3. for the general formula.
[6] See Appendix 1 paragraph 4. for the general formula.
[7] See Appendix 1 paragraph 5. for the general formula.
[8] See Appendix 1 paragraph 6. for the general formula.
[8] See Appendix 1 paragraph 7. for the general formula.

in column[10]. To get the person years lived without disability (Table 1.3 column[11]) we multiply the person years lived in the age interval (column[7]) by the proportion of people without disability at that age (1- column[10]). If the survey has a complex design or if the non-response rates are high in the initial sample, column[10] should contain weighted rates, that is to say prevalence rates adjusted for the design characteristics and for the bias introduced in the final sample by the non-response profiles.

8. The total number of years lived without disability (column[12]) are found from column[11] in the same way as in paragraph 5 above.

9. Similarly the DFLE at each age (column[13]) is found in the same way as paragraph 6, from column[12] divided by column[5]. Thus a women aged 80 years in 1988 could expect to live 8.16 years of which 4.89 years (59.9%) would be free of disability.

Table 1.1: Data for the calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single year life table

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
Age	Mid year population (1000s)	No. deaths (1000s)							Proportion with disability			
x	P_x	D_x							π_x			
75	224.4	8.429							0.140			
76	211.9	8.979							0.140			
77	202.3	9.342							0.140			
78	195.0	9.872							0.140			
79	186.0	10.542							0.140			
80	172.5	10.896							0.251			
81	156.3	11.200							0.251			
82	143.5	11.490							0.251			
83	131.3	11.633							0.251			
84	115.1	11.283							0.251			
85	101.6	11.240							0.559			
86	86.2	10.718							0.559			
87	76.4	10.080							0.559			
88	63.9	9.319							0.559			
89	49.3	8.234							0.559			
90	163.8	37.645							0.559			

Table 1.2:   Calculation of the life table quantities for DFLE by the Sullivan method using a single year life table

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
Age	Mid-year population (1000s)	No. deaths (1000s)	Central Death rate	Conditional probability of death	Probability of survival to age x	Person years lived in age interval	Total number of years lived from age x	Total Life Expectancy	Proportion with disability			
x	P_x	D_x	M_x	Q_x	ℓ_x	L_x	T_x	e_x	π_x			
75	224.4	8.429	0.038	0.037	1.000	0.982	10.963	**10.963**	0.140			
76	211.9	8.979	0.042	0.041	0.963	0.943	9.982	**10.364**	0.140			
77	202.3	9.342	0.046	0.045	0.923	0.902	9.039	**9.791**	0.140			
78	195.0	9.872	0.051	0.049	0.881	0.860	8.136	**9.230**	0.140			
79	186.0	10.542	0.057	0.055	0.838	0.815	7.277	**8.684**	0.140			
80	172.5	10.896	0.063	0.061	0.792	0.768	6.462	**8.161**	0.251			
81	156.3	11.200	0.072	0.069	0.743	0.718	5.694	**7.661**	0.251			
82	143.5	11.490	0.080	0.077	0.692	0.665	4.977	**7.193**	0.251			
83	131.3	11.633	0.089	0.085	0.639	0.612	4.311	**6.751**	0.251			
84	115.1	11.283	0.098	0.093	0.584	0.557	3.700	**6.331**	0.251			
85	101.6	11.240	0.111	0.105	0.530	0.502	3.143	**5.932**	0.559			
86	86.2	10.718	0.124	0.117	0.474	0.447	2.641	**5.568**	0.559			
87	76.4	10.080	0.132	0.124	0.419	0.393	2.194	**5.240**	0.559			
88	63.9	9.319	0.146	0.136	0.367	0.342	1.801	**4.909**	0.559			
89	49.3	8.234	0.167	0.154	0.317	0.293	1.459	**4.603**	0.559			
90	163.8	37.645	0.230	1.000	0.268	1.167	1.167	**4.351**	0.559			

Table 1.3:   Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single year life table

[1] Age	[2] *Mid-year population (1000s)*	[3] *No. deaths (1000s)*	[4] Central Death rate	[5] Conditional probability of death	[6] Probability of survival to age x	[7] Person years lived in age interval	[8] Total number of years lived from age x	[9] Total Life Expectancy	[10] *Proportion with disability*	[11] Person years lived without disability in interval	[12] Total years lived without disability from age x	[13] Disability-free life expectancy
x	P_x	D_x	M_x	Q_x	ℓ_x	L_x	T_x	e_x	π_x	$[1-\pi_x]L_x$	$\Sigma[1-\pi_x]L_x$	$DFLE_x$
75	*224.4*	*8.429*	0.038	0.037	1.000	0.982	10.963	**10.963**	*0.140*	0.844	7.743	**7.743**
76	*211.9*	*8.979*	0.042	0.041	0.963	0.943	9.982	**10.364**	*0.140*	0.811	6.899	**7.163**
77	*202.3*	*9.342*	0.046	0.045	0.923	0.902	9.039	**9.791**	*0.140*	0.776	6.088	**6.595**
78	*195.0*	*9.872*	0.051	0.049	0.881	0.860	8.136	**9.230**	*0.140*	0.739	5.312	**6.026**
79	*186.0*	*10.542*	0.057	0.055	0.838	0.815	7.277	**8.684**	*0.140*	0.701	4.573	**5.457**
80	*172.5*	*10.896*	0.063	0.061	0.792	0.768	6.462	**8.161**	*0.251*	0.575	3.872	**4.890**
81	*156.3*	*11.200*	0.072	0.069	0.743	0.718	5.694	**7.661**	*0.251*	0.537	3.297	**4.436**
82	*143.5*	*11.490*	0.080	0.077	0.692	0.665	4.977	**7.193**	*0.251*	0.498	2.760	**3.988**
83	*131.3*	*11.633*	0.089	0.085	0.639	0.612	4.311	**6.751**	*0.251*	0.458	2.261	**3.541**
84	*115.1*	*11.283*	0.098	0.093	0.584	0.557	3.700	**6.331**	*0.251*	0.417	1.803	**3.085**
85	*101.6*	*11.240*	0.111	0.105	0.530	0.502	3.143	**5.932**	*0.559*	0.221	1.386	**2.616**
86	*86.2*	*10.718*	0.124	0.117	0.474	0.447	2.641	**5.568**	*0.559*	0.197	1.165	**2.455**
87	*76.4*	*10.080*	0.132	0.124	0.419	0.393	2.194	**5.240**	*0.559*	0.173	0.968	**2.311**
88	*63.9*	*9.319*	0.146	0.136	0.367	0.342	1.801	**4.909**	*0.559*	0.151	0.794	**2.165**
89	*49.3*	*8.234*	0.167	0.154	0.317	0.293	1.459	**4.603**	*0.559*	0.129	0.644	**2.030**
90	*163.8*	*37.645*	0.230	*1.000*	0.268	1.167	1.167	**4.351**	*0.559*	0.515	0.515	**1.919**

Sullivan health expectancy using an abridged life table: Example 2

Why do we need to use an abridged life table where instead of taking single years of age, we group the ages into five or ten year bands? Before spreadsheets became accessible, calculation of life tables by single years of age was tedious and time consuming. With the advent of spreadsheets, one can argue that abridged life tables are unnecessary. However, Example 1 demonstrated that the prevalence of disability (or other health state) by single years of age may show considerable fluctuations due to sampling variation. Thus it may be preferable to use the average prevalence ratios for five or ten years age intervals. Instead of the method in Example 1, with the life table in single years of age, it is possible to calculate the Sullivan health expectancy with an abridged life table, where the age intervals are greater than one year.

We will now show an example of such a calculation of disability-free life expectancy for French females. The prevalence of disability for females in France was obtained from a disability survey conducted in 1991. The survey covered only the non-institutionalized population but for simplicity here we use previously combined prevalences from both non-institutionalized and institutionalized populations (see Example 5 later which explains how to calculate the combined prevalence for the non-institutionalized and institutionalized populations).

The method below calculates the central death rates from the deaths and populations of the ages groups. We take 5 year age groups apart from the final open-ended group of 85+ which we assume is of length 10 years. In combining single years of age into age groups we are introducing a bias although we are increasing the precision of our estimates of the survival curve (and prevalence). However we can obtain some information which allows us to compensate for the bias. In Example 4 we will discuss a second method of calculating health expectancy with an abridged life table.

Calculation of the life table and life expectancy
Since much of the calculation in the present example is the same as Example 1, the column numbers in Tables 2.1-2.2 relate to the associated columns in Tables 1.1-1.3.

1. Defining the length of the age group as n, we begin as in Example 1 with the calculation of the central death rates $_nM_x$ (column[4]) by dividing the deaths (column[3]) by the mid-year population (column[2]) since again both the deaths and population are based on one calendar year[9]. The values are shown in Table 2.1.

As alluded to earlier, grouping up the ages introduces bias into the estimate of the survival curve but gives greater precision. We introduce the column a_x which gives us a picture of the curvature of the true survival curve over the age interval (Chiang, 1984). In Example 1 we assumed that a_x = 0.5 since for single years of age the assumption that the survival curve can be approximated as a straight line between each year of age is not unreasonable. As we use age groups of five or more years this assumption is less reasonable and values of a_x below 0.5 indicate that the survival curve lies below a straight line over the age interval whilst values of a_x greater than 0.5 indicate that the true curve lies above the straight line. Values of a_x are sometimes available with life tables produced by National Statistical Offices or can be obtained from the World Health Organization.

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[9] See Appendix paragraph 8. for the general formula.

Table 2.2 shows the results of the calculation of the probabilities of death in the age interval conditional on having survived to the beginning of the interval, $_nQ_x$ (column[5]). Again, as in Example 1, the final entry in column[5] corresponding to 85+ years is, by definition, 1.0. The other entries are calculated as $5*\text{column}[4]/(1 + (1-a_x)*5*\text{column}[4])^{10}$.

3. Column[6] shows the probability of survival (l_x) to the beginning of the age interval. The first entry is by definition 1.0 whilst each successive row is calculated in the same way as Example 1, as the previous entry multiplied by its corresponding (1-column[5]).

4. Column[7] shows the person years lived **in each age interval** $_nL_x$. This is found by multiplying the entry in column[6] for the same age group by n*a_x and adding this to n*(1-a_x) multiplied by the entry in column[6] for the next age group, where n is the size of the age group. In our example n=10 for the final age group and 5 for the remainder. Thus for the person years lived in the 30-34 year age group, $_5L_{30}$, we have n=5, a_x=0.538, l_{30}=0.9840 and l_{35}=0.9805. Thus $_5L_{30}$= 5*0.538*0.9840 + 5*0.462*0.9805.

5. The calculation of the total number of years lived (column[8]) and the total life expectancy (column[9]) continue in exactly the same way as Example 1, steps 5 and 6.

Calculation of Disability-Free Life Expectancy (DFLE)
The prevalence of disability in column[10] is expressed as a percentage in this example but is used in the calculations in the same way as Example 1. Thus 6.4% = 0.064.

6. The calculation of columns [11]-[13] continue as for Example 1 steps 7-9.

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[10] See Appendix 1 paragraph 8. for the general formula.

Table 2.1: **Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using an abridged life table**

	[1]		[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
	Age group		*Mid-year population*	*No. deaths*	Central Death rate									
x	-	x+n	P_x	D_x	$_nM_x$									
0	-	4	*1842444*	*2802*	0.001521									
5	-	9	*1873804*	*313*	0.000167									
10	-	14	*1866802*	*301*	0.000161									
15	-	19	*2025249*	*691*	0.000341									
20	-	24	*2120295*	*975*	0.000460									
25	-	29	*2163219*	*1255*	0.000580									
30	-	34	*2158003*	*1534*	0.000711									
35	-	39	*2137312*	*2028*	0.000949									
40	-	44	*2157647*	*3120*	0.001446									
45	-	49	*1531927*	*3274*	0.002137									
50	-	54	*1412383*	*4312*	0.003053									
55	-	59	*1525871*	*6747*	0.004422									
60	-	64	*1544558*	*10077*	0.006524									
65	-	69	*1479532*	*14908*	0.010076									
70	-	74	*974049*	*16574*	0.017016									
75	-	79	*954451*	*29857*	0.031282									
80	-	84	*819903*	*51059*	0.062274									
85	-	+	*684460*	*114915*	0.167891									

Table 2.2: **Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using an abridged life table**

[1] Age group	[2] Mid-year population	[3] No. deaths	[4] Central Death rate		[5] Conditional probability of death in age interval	[6] Probability of survival to age x	[7] Person years lived in age interval	[8] Total number of years lived from age x	[9] Total Life Expectancy	[10] Proportion of age group with disability	[11] Person years lived without disability in interval	[12] Total years lived without disability from age x	[13] Disability-free life expectancy
x - x+n	P_x	D_x	$_nM_x$	a_x	$_nQ_x$	l_x	$_nL_x$	T_x	e_x	π_x	$(1-\pi_x)*L_x$	T_x	$DFLE_x$
0 - 4	1842444	2802	0.001521	0.178	0.00755678	1.0000	4.969	81.057	**81.057**	1.0%	4.919	70.111	**70.111**
5 - 9	1873804	313	0.000167	0.477	0.00083483	0.9924	4.960	76.088	**76.667**	2.4%	4.841	65.192	**65.688**
10 - 14	1866802	301	0.000161	0.530	0.00080589	0.9916	4.956	71.128	**71.729**	2.6%	4.827	60.351	**60.861**
15 - 19	2025249	691	0.000341	0.555	0.00170467	0.9908	4.950	66.172	**66.785**	4.3%	4.737	55.524	**56.038**
20 - 24	2120295	975	0.000460	0.517	0.00229666	0.9891	4.940	61.221	**61.894**	3.7%	4.757	50.786	**51.345**
25 - 29	2163219	1255	0.000580	0.519	0.00289673	0.9869	4.927	56.281	**57.031**	6.4%	4.612	46.029	**46.642**
30 - 34	2158003	1534	0.000711	0.538	0.00354838	0.9840	4.912	51.354	**52.189**	4.0%	4.715	41.417	**42.090**
35 - 39	2137312	2028	0.000949	0.524	0.00473358	0.9805	4.891	46.442	**47.365**	5.6%	4.618	36.701	**37.431**
40 - 44	2157647	3120	0.001446	0.524	0.00720532	0.9759	4.863	41.551	**42.578**	6.4%	4.551	32.084	**32.877**
45 - 49	1531927	3274	0.002137	0.528	0.01063226	0.9688	4.820	36.688	**37.868**	8.8%	4.396	27.532	**28.418**
50 - 54	1412383	4312	0.003053	0.531	0.01515654	0.9585	4.759	31.868	**33.247**	10.9%	4.240	23.137	**24.138**
55 - 59	1525871	6747	0.004422	0.534	0.02188313	0.9440	4.672	27.109	**28.718**	12.6%	4.083	18.897	**20.018**
60 - 64	1544558	10077	0.006524	0.534	0.03213211	0.9233	4.548	22.438	**24.300**	15.6%	3.838	14.814	**16.043**
65 - 69	1479532	14908	0.010076	0.534	0.04922552	0.8937	4.366	17.890	**20.019**	19.6%	3.510	10.976	**12.281**
70 - 74	974049	16574	0.017016	0.539	0.08186437	0.8497	4.088	13.524	**15.917**	28.6%	2.919	7.465	**8.786**
75 - 79	954451	29857	0.031282	0.543	0.14596476	0.7801	3.640	9.436	**12.096**	37.0%	2.293	4.547	**5.828**
80 - 84	819903	51059	0.062274	0.529	0.27156185	0.6663	2.905	5.796	**8.699**	51.3%	1.415	2.253	**3.382**
85 - +	684460	114915	0.167891	0.596	1.00000000	0.4853	2.891	2.891	**5.956**	71.0%	0.838	0.838	**1.727**

The standard error of the Sullivan health expectancy: Example 3

The prevalence of disability by single or five-year age groups show considerable fluctuation due to sampling variation. Mortality rates are also subject to random variation. Since the Sullivan health expectancy combines such mortality and morbidity rates, it too is subject to random variation. To assess the size of this random variation, we shall calculate the standard error of DFLE for females in France in 1991. These continue from the calculations in Example 2 (shown in Table 2.2) and the column numbers for the first 6 columns in Table 3 correspond to those in Table 2.2. For clarity we omit from Table 3 columns [2] - [5].

1. The first extra column (column[14]) in Table 3 is the number of persons in the age interval who took part in the survey. If the survey does not have a complex design and if the non response rates are not high, then column[14] contains the denominators used to calculate the prevalence in column[10].

2. Column[15] = {column[10]*(1-column[10])}/column[14][11]. This is a general formula to estimate the variance of the prevalence rates. However, if the survey providing the prevalence rates has a complex sampling design then more accurate estimates of the variances of the prevalence rates will be provided by the appropriate Statistical Institute involved in the survey. In case these more accurate estimates are not available, a simple approximation to the calculation of the standard error of DFLE is to use the general formula above but with the weighted prevalence in column[10] and in column[14] the unweighted number of persons in the age interval i.e. the <u>actual</u> number who took part in the survey in the given age interval.

3. Column[16] = column[7]*column[7]*column[15][12].

4. Column[17] is found by summing the entries in column[16] from that age interval to the final age interval (in this case 85+).

5. The variance of the health expectancy is given in column[18] where column[18] = column[17]/(column[6]*column [6]).

6. The standard error of the health expectancy in column[19] is the square root of column[18].

If required, approximate 95% confidence intervals for the DFLE are given by column[13] - 1.96*column[19] and column[13]+1.96*column[19].

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[11] See Appendix 1 paragraph 9. for the general formula.

Table 3: **Calculation of the standard error for DFLE by the Sullivan method using an abridged life table**

[1] Age group	[6] Probability of survival to age x	[7] Person years lived in age interval	[8] Total number of years lived from age x	[9] Total Life Expectancy	[10] *Proportion of age group with disability*	[11] Person years lived without disability in interval	[12] Total years lived without disability from age x	[13] Disability-free life expectancy	[14] Number in survey in age interval	[15]	[16]	[17]	[18]	[19] Standard error of $DFLE_x$
x - x+n	l_x	$_nL_x$	T_x	e_x	π_x	$(1-\pi_x)*L_x$	T_x	$DFLE_x$	N_x	$S^2(\pi_x)$	$L^2S^2(\pi_x)$	$\Sigma L^2S^2(\pi_x)$	$S^2(DFLE_x)$	$S(DFLE_x)$
0 - 4	1.0000	4.969	81.057	**81.057**	*1.0%*	4.919	70.111	**70.111**	772	0.000013	0.000317	0.06919	0.06919	**0.263**
5 - 9	0.9924	4.960	76.088	**76.667**	*2.4%*	4.841	65.192	**65.688**	741	0.000032	0.000778	0.06887	0.06992	0.264
10 - 14	0.9916	4.956	71.128	**71.729**	*2.6%*	4.827	60.351	**60.861**	745	0.000034	0.000835	0.06809	0.06925	0.263
15 - 19	0.9908	4.950	66.172	**66.785**	*4.3%*	4.737	55.524	**56.038**	775	0.000053	0.001301	0.06726	0.06851	0.262
20 - 24	0.9891	4.940	61.221	**61.894**	*3.7%*	4.757	50.786	**51.345**	728	0.000049	0.001194	0.06596	0.06742	0.260
25 - 29	0.9869	4.927	56.281	**57.031**	*6.4%*	4.612	46.029	**46.642**	805	0.000074	0.001807	0.06476	0.06650	0.258
30 - 34	0.9840	4.912	51.354	**52.189**	*4.0%*	4.715	41.417	**42.090**	892	0.000043	0.001039	0.06296	0.06502	0.255
35 - 39	0.9805	4.891	46.442	**47.365**	*5.6%*	4.618	36.701	**37.431**	814	0.000065	0.001554	0.06192	0.06440	0.254
40 - 44	0.9759	4.863	41.551	**42.578**	*6.4%*	4.551	32.084	**32.877**	824	0.000073	0.001719	0.06036	0.06339	0.252
45 - 49	0.9688	4.820	36.688	**37.868**	*8.8%*	4.396	27.532	**28.418**	627	0.000128	0.002974	0.05864	0.06248	0.250
50 - 54	0.9585	4.759	31.868	**33.247**	*10.9%*	4.240	23.137	**24.138**	571	0.000170	0.003851	0.05567	0.06059	0.246
55 - 59	0.9440	4.672	27.109	**28.718**	*12.6%*	4.083	18.897	**20.018**	603	0.000183	0.003986	0.05182	0.05815	0.241
60 - 64	0.9233	4.548	22.438	**24.300**	*15.6%*	3.838	14.814	**16.043**	611	0.000215	0.004456	0.04783	0.05610	0.237
65 - 69	0.8937	4.366	17.890	**20.019**	*19.6%*	3.510	10.976	**12.281**	534	0.000295	0.005625	0.04338	0.05431	0.233
70 - 74	0.8497	4.088	13.524	**15.917**	*28.6%*	2.919	7.465	**8.786**	380	0.000537	0.008980	0.03775	0.05229	0.229
75 - 79	0.7801	3.640	9.436	**12.096**	*37.0%*	2.293	4.547	**5.828**	310	0.000752	0.009964	0.02877	0.04728	0.217
80 - 84	0.6663	2.905	5.796	**8.699**	*51.3%*	1.415	2.253	**3.382**	228	0.001096	0.009249	0.01881	0.04237	0.206
85 - +	0.4853	2.891	2.891	**5.956**	*71.0%*	0.838	0.838	**1.727**	180	0.001144	0.009558	0.00956	0.04058	0.201

Testing the equality of two Sullivan health expectancies : Example 4

To test whether two Sullivan health expectancies are equal, we need to calculate the variance (and standard error) of each of the health expectancies. We shall compare the Disability-Free Life Expectancies (DFLE) for females in France in 1981 and 1991.

The test of the hypothesis of equality of DFLE for females in 1981 and 1991 is shown in Table 4.

7. Column[2] and column[3] of Table 4 are the DFLE for females in France in 1991 and its standard error. These were already calculated in column[13] and column[19] in Table 3. The equivalent values for 1981 are shown in column[4] and column[5] of Table 4.

8. Column[6] shows the difference between the DFLE for the two years and is found by subtracting column[4] from column[2].

9. The <u>approximate</u> standard error of this difference is shown in column[7] and is found by adding together column[3] and column[5][12].

10. The z-statistic in column[8] is formed from column[6]/column[7]. If a one-sided test is required e.g. testing that, at a particular age, $DFLE_{1991} > DFLE_{1981}$ then the p-value (column[9]) is the probability that a standard normal variate exceeds the value in column[8]. If a two-sided test is required, simply testing that a difference exists between the DFLE at each time then the p-value (column[9]) is twice the probability that a standard normal variate exceeds the value in column[8]. These probabilities are found from tables of the standard normal distribution (see Appendix 2).

The results show that Disability-Free Life Expectancy (DFLE) for females in France had significantly increased between 1981 and 1991.

There are two other issues worth discussing in this example because they are particularly relevant for comparisons between times or geographical areas.

Firstly, especially with national health surveys, the numbers of people at the oldest ages may well be small. Thus a large final age group is required to give sufficient precision to estimates for this age group. However, the prevalence of disability increases exponentially with age and therefore assuming a constant prevalence for this large open-ended age group will produce biased estimates. Fortunately, if we are comparing two points in time or two geographical areas, the same bias will apply to both. Thus estimates of the **difference** in DFLE will be unbiased.

Secondly, if estimates of DFLE for different regions in a country are to be compared, then the numbers in many age groups may be small within regions. To account for this we may wish to create an abridged life table with wider than 5 year age groups, perhaps say 15 or 20 year age groups i.e. 0-14, 15-24, 25-44,45-64, 65+. Another method of forming the abridged life table than that described in Example 2 is to calculated the L_x for the abridged table by summing values for single years of age (or smaller age groups). Hence L_{15-24} can be calculated as

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[12] See Appendix 1 paragraph 10. for the general formula.

$$L_{15-24} = L_{15} + L_{16} + ... + L_{24} \quad \text{or} \quad L_{15-24} = L_{15-19} + L_{20-24}.$$

Again, although assuming a constant prevalence over large age groups will introduce a bias, the bias will be present for all regions therefore making the differences in DFLE between regions unbiased.

Table 4: Comparisons of Disability-Free Life Expectancy in 1981 ($DFLE_{(1)}$) and 1991 ($DFLE_{(2)}$) for females in France

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
Age group	Disability-free life expectancy 1991	Standard error of DFLE 1991	Disability-free life expectancy 1981	Standard error of DFLE 1981	Difference in DFLE between 1981 and 1991	Approximate standard error of difference in DFLE between 1981 and 1991	z statistic	p value Pr(Z≥z)
x - x+n	$DFLE_{(2)}$	$S(DFLE_{(2)})$	$DFLE_{(1)}$	$S(DFLE_{(1)})$	$DFLE_{(2)} - DFLE_{(1)}$	$S(DFLE_{(1)}) + S(DFLE_{(2)})$	z	p
0- 4	70.111	0.263	67.772	0.232	2.339	0.495	4.73	<0.0001
5- 9	65.688	0.264	63.484	0.234	2.204	0.498	4.43	<0.0001
10-14	60.861	0.263	58.633	0.233	2.228	0.496	4.49	<0.0001
15-19	56.038	0.262	53.782	0.232	2.256	0.494	4.57	<0.0001
20-24	51.345	0.26	49.029	0.231	2.316	0.491	4.72	<0.0001
25-29	46.642	0.258	44.227	0.231	2.415	0.489	4.94	<0.0001
30-34	42.09	0.255	39.503	0.23	2.587	0.485	5.33	<0.0001
35-39	37.431	0.254	34.783	0.229	2.648	0.483	5.48	<0.0001
40-44	32.877	0.252	30.143	0.228	2.734	0.480	5.70	<0.0001
45-49	28.418	0.25	25.679	0.226	2.739	0.476	5.75	<0.0001
50-54	24.138	0.246	21.326	0.224	2.812	0.470	5.98	<0.0001
55-59	20.018	0.241	17.122	0.22	2.896	0.461	6.28	<0.0001
60-64	16.043	0.237	13.293	0.214	2.750	0.451	6.10	<0.0001
65-69	12.281	0.233	9.884	0.207	2.397	0.440	5.45	<0.0001
70-74	8.786	0.229	6.857	0.198	1.929	0.427	4.52	<0.0001
75-79	5.828	0.217	4.246	0.183	1.582	0.400	3.96	<0.0001
80-84	3.382	0.206	2.344	0.162	1.038	0.368	2.82	<0.01
85+	1.727	0.201	1.507	0.153	0.220	0.354	0.62	>0.20

Sullivan health expectancy using separate sources of data for institutionalized and non-institutionalized : Example 5

Since rates of institutionalization differ both between countries and within countries over time, using surveys which omit those in institutions for the calculation of prevalence of health states for the Sullivan indicator can seriously bias comparisons. We now demonstrate how information on the institutionalized population (who were omitted from the French health survey) used in the previous examples can be incorporated to produce a prevalence for the total French population.

The most common method to incorporate the institutionalized population into the health expectancy estimates requires data on the proportion of the total population who are in institutions by age group. These data are usually obtained from Census figures. The assumption made is that all those in institutions are in a particular health state (in our example we assume they all have disability).

1. We show in Table 5 columns[6], [7] and [9] from Example 2 as these will be required later to show the changes to the calculation of the standard error of DFLE.

2. We rename the proportion with disability from the survey (non-institutionalized only) to be π'_x and its column to be column[20]. The new proportion disabled in the total population (non-institutionalized and institutionalized) will be given in column[10] to correspond with column numbers in Example 2. In column[14] we again have the number of people interviewed in the survey in each age group, N_x.

3. The proportion in institutions, obtained from Census figures, is I_x (column[21]).

4. The new proportion disabled in the total population is calculated in column[10] by Column[10]=(1-column[21])*column[20] + column[21][13]. The remainder of the calculation of DFLE continues as in Example 1 steps 7-9.

5. A further amendment is required in the calculation of the standard error of DFLE. In Example 3 step 2 we calculated the variance of the prevalence rates. This step, to calculate column[15] in Table 3 should be replaced by Column[15]=(1-column[21])*(1-column[21])*column[20]*(1-column[20])/column[14][14]. You will notice that the figures in Table 5 are slightly different to those in Example 3 Table 3 as, for simplicity, we did not account for the fact that the prevalence was a combined one from institutionalized and non-institutionalized persons.

The calculation of the standard error of DFLE continues in the same way as Example 3 from step 3 onward.

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[13] See Appendix 1 paragraph 11. for the general formula.
[14] See Appendix 1 paragraph 12. for the general formula.

Table 7: Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using an abridged life table with institutionalized population

[1] Age group x - x+n	[6] Number of survivors entering age group x ℓ_x	[7] Person years lived in age interval L_x	[9] Total Life Expectancy e_x	[3] Proportion in survey with disability π'_x	[14] Number in survey in age interval N_x	[2] Proportion in institutions I_x	[10] Proportion in total population with disability π_x	[15] $S^2(\pi_x)$
0- 4	1.0000	4.969	81.057	0.010	772	0.000	**0.01**	**0.000013**
5- 9	0.9924	4.960	76.667	0.024	741	0.000	**0.024**	**0.000032**
10-14	0.9916	4.956	71.729	0.026	745	0.000	**0.026**	**0.000034**
15-19	0.9908	4.950	66.785	0.042	775	0.001	**0.043**	**0.000052**
20-24	0.9891	4.940	61.894	0.036	728	0.001	**0.037**	**0.000048**
25-29	0.9869	4.927	57.031	0.063	805	0.001	**0.064**	**0.000073**
30-34	0.9840	4.912	52.189	0.039	892	0.001	**0.04**	**0.000042**
35-39	0.9805	4.891	47.365	0.055	814	0.001	**0.056**	**0.000064**
40-44	0.9759	4.863	42.578	0.063	824	0.001	**0.064**	**0.000071**
45-49	0.9688	4.820	37.868	0.086	627	0.002	**0.088**	**0.000125**
50-54	0.9585	4.759	33.247	0.106	571	0.003	**0.109**	**0.000165**
55-59	0.9440	4.672	28.718	0.122	603	0.004	**0.126**	**0.000176**
60-64	0.9233	4.548	24.300	0.151	611	0.006	**0.156**	**0.000207**
65-69	0.8937	4.366	20.019	0.188	534	0.010	**0.196**	**0.000280**
70-74	0.8497	4.088	15.917	0.274	380	0.016	**0.286**	**0.000507**
75-79	0.7801	3.640	12.096	0.345	310	0.038	**0.37**	**0.000675**
80-84	0.6663	2.905	8.699	0.468	228	0.085	**0.513**	**0.000914**
85+	0.4853	2.891	5.956	0.632	180	0.211	**0.71**	**0.000804**

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Appendix 1

Here we give further technical details of the calculation of health expectancy by the Sullivan method. Since the basis of these calculations are quantities derived from life tables Chiang (1984) provides further explanations.

Sullivan health expectancy with a complete life table
Column numbers refer to those in Table 1 - 3.

1. Let the age intervals (*column [1]*) for the calculations be [*x, x+1*[. Let *W* be the last interval [*W, ∞*[and *x=0,1,...,W*.

2. Column [4] gives the age-specific death rates $M_x = D_x/(P_x \cdot T)$ where D_x is the number of deaths at age *x* registered during the period of length *T* and P_x is the average population exposed to risk. In the example *T* is one calendar year and P_x is the midyear population for the calendar year (or average of the calendar years) corresponding to the survey in which the prevalence of the health state is estimated.

3. Column [5] gives q_x, the proportion of those alive at age x who die in the interval [*x,x+1*[. These are derived from the corresponding age-specific deaths rates of the current population, using the formula:

$$q_x = \frac{M_x}{1+(1-a_x)M_x} \qquad x=0,1...,W.$$

   Each of those people who die in the interval [*x, x+1*[have lived *x* complete years plus a fraction, a_x, of the last year. a_x is often considered to be 0.5 on average (assuming that deaths are uniformly distributed in the interval, except for the first four years of life (ages 1 to 4 years). However, the result is close enough, certainly for single year age groups, to reality to allow the approximation for these age groups also.

4. We begin the calculation of the number of survivors at age *x*, ℓ_x (column[6]), by assuming a radix population ℓ_0 of 100,000. For the remaining entries in column [6] we use the equation $\ell_{x+1} = \ell_x(1-q_x)$ for *x=0,1,...,W-1*.

5. The total number of years lived by the cohort in the interval [*x, x+1*[is given by L_x (column[7]). Apart from the last age interval W (see 6. below), the members of the cohort who survive the interval (ℓ_{x+1}) will each contribute one year (since the age intervals are one year). The remainder ($\ell_x - \ell_{x+1}$) will each contribute a fraction a_x. Hence for *x=0,1,...W-1*

$$L_x = \ell_{x+1} + a_x(\ell_x - \ell_{x+1})$$
$$= (1-a_x)\ell_{x+1} + a_x\ell_x$$
$$= \frac{(\ell_{x+1} + \ell_x)}{2} \text{ if } a_x = 0.5$$

6. Often life tables are made up to age 100 years as little information is available after this age. For the last age interval *W* we use the formula:

$$L_W = \frac{\ell_W}{M_W}$$

Quantification of mortality above the age of 80 years and evaluation of survival at these ages raise substantial problems. A number of methodological and practical difficulties are involved. Errors in data play a much more serious role in the case of the elderly than in other groups of the population and errors are present in both the death and population statistics (Kannisto, 1988). However, the biases are generally small compared to the standard errors and can safely be ignored in most cases.

7. The life expectancy at age x, e_x (column[9]), is given by

$$e_x = \frac{1}{\ell_x} \sum_{i=x}^{W} L_i$$

The calculation of health expectancy follows similar lines. If we assume two states called Disability-free (DF) and with disability (D) then the Disability-Free Life Expectancy at age x ($DFLE_x$) and the life expectancy with disability (DLE_x) are defined by:

$$DFLE_x = \frac{1}{\ell_x} \sum_{i=x}^{W} L_i(DF) \quad \text{and}$$

$$DLE_x = \frac{1}{\ell_x} \sum_{i=x}^{W} L_i(D)$$

where $L_i(DF)$ and $L_i(D)$ are the number of person years lived from age x onwards in the states DF (without disability) and D (with disability) respectively. Using the Sullivan method as an approximation of health expectancy (multistate) leads to the hypothesis that

$$L_i(D) = \pi_i L_i \qquad i=0,...,W,$$

where π_i is the prevalence of disability at age I. Thus for $x=0,...,W$

$$DFLE_x = \frac{1}{\ell_x} \sum_{i=x}^{W} (1-\pi_i) L_i \quad \text{and}$$

$$DLE_x = \frac{1}{\ell_x} \sum_{i=x}^{W} \pi_i L_i \quad .$$

Amendments for an abridged life table
8. Assume the length of the age interval is no longer one year but n (this may vary for different age groups but the subscript is omitted for clarity). Then for the age interval $[x,x+n[$:

$$_nM_x = \frac{_nD_x}{_nP_x \times T} \qquad x=0,1,...,W,$$

$$q_x = \frac{n\,_nM_x}{1+n(1-a_x)\,_nM_x} \qquad x=0,1...,W.$$

and

$$_nL_x = n\ell_{x+1} + na_x(\ell_x - \ell_{x+1})$$
$$= n(1-a_x)\ell_{x+1} + na_x\ell_x$$

Calculation of variance of Sullivan health expectancy

The same notation is taken as above and the general formulae shown for use with an abridged life table with ages x, $x+n$,... and $x=0,...,W$.

9. Mathers (1991) shows that the variances of $DFLE_x$ and DLE_x can be approximated by:

$$S^2(DLE_x) = \frac{1}{\ell_x^2}\sum_{x=0}^{W-1}\ell_x^2[(1-a_x)n\pi_x + DLE_{x+n}]^2 S^2(p_x) + \frac{1}{\ell_x^2}\sum_{x=0}^{W}L_x^2 S^2(\pi_x)$$

$$S^2(DFLE_x) = \frac{1}{\ell_x^2}\sum_{x=0}^{W-1}\ell_x^2[(1-a_x)n(1-\pi_x) + DFLE_{x+n}]^2 S^2(p_x) + \frac{1}{\ell_x^2}\sum_{x=0}^{W}L_x^2 S^2(1-\pi_x)$$

where $p_x = \ell_{x+n}/\ell_x$ and a_x, as before, is the fraction of the interval lived by those who die in the interval.

If the sample size of the survey producing the prevalence ratios is not very large compared to the population on which the mortality data are based then $S^2(p_x)$ is negligible and the first terms of the above equations can be ignored (Newman, 1988).

In general $S^2(\pi_x) = S^2(1-\pi_x)$ and both can be approximated by

$$S^2(\pi_x) = \frac{\pi_x(1-\pi_x)}{N_x}$$

where N_x is the number of persons in the age interval $[x,x+n[$ participating in the prevalence survey. However it may be necessary if the survey has a complex sampling design (eg stratified or cluster) this will have to be included in this term. A simple approximation in the case of complex sampling design is to use the weighted π_x and the unweighted N_x.

Hence the variances of $DFLE_x$ and DLE_x can be approximated by

$$S^2(DLE_x) \approx \frac{1}{\ell_x^2}\sum_{x=0}^{W}L_x^2 S^2(\pi_x) \approx \frac{1}{\ell_x^2}\sum_{x=0}^{W}L_x^2 \frac{\pi_x(1-\pi_x)}{N_x}$$

and

$$S^2(DFLE_x) \approx \frac{1}{\ell_x^2}\sum_{x=0}^{W}L_x^2 S^2(1-\pi_x) \approx \frac{1}{\ell_x^2}\sum_{x=0}^{W}L_x^2 \frac{\pi_x(1-\pi_x)}{N_x}$$

10. Since the estimates of health expectancy are the means of random variables assumed to be independent, application of the central limit theorem means that they can be assumed to have normal distributions. Therefore the hypothesis of equality of two health expectancies, say $DFLE_{(1)}$ and $DFLE_{(2)}$, may be tested by the following Z-score:

$$Z = \frac{DFLE_{(1)} - DFLE_{(2)}}{\sqrt{S^2(DFLE_{(1)} - DFLE_{(2)})}}$$

If we denote by $S(DFLE_{(1)})$ and $S(DFLE_{(2)})$ the standard errors of $DFLE_{(1)}$ and $DFLE_{(2)}$ respectively then since

$$S^2(DFLE_{(1)} - DFLE_{(2)}) \le [S(DFLE_{(1)}) + S(DFLE_{(2)})]^2$$

then we compute the approximate Z-score (a conservative approximation) as

$$Z = \frac{DFLE_{(1)} - DFLE_{(2)}}{S(DFLE_{(1)}) + S(DFLE_{(2)})}$$

The hypothesis of equality is rejected if the absolute value of the Z-score is too large, say ≥ 1.96 (see Appendix 2 for critical values of the Z-score).

Incorporating data from separate surveys of community-living and institutionalized

11. We denote by π'_x the proportion with disability estimated from the survey and I_x the proportion in institutions in the age interval x to x+n. Then π_x the proportion disabled or in institutions is given by

$$\pi_x = (1 - I_x)\,\pi'_x + I_x .$$

12. Since I_x is derived from Census data we do not consider it to be a random variable. Therefore $S^2(\pi_x)$ can be approximated by

$$S^2(\pi_x) = (1 - I_x)^2 \frac{\pi'_x(1 - \pi'_x)}{N_x}$$

Appendix 2

Table 8: Critical values of the z-statistic

z critical values	Level of significance for a *two-*tailed test	Level of significance for a *one-*tailed test
1.28	0.20	0.10
1.645	0.10	0.05
1.96	0.05	0.025
2.33	0.02	0.01
2.58	0.01	0.005
3.09	0.002	0.001
3.29	0.001	0.0005